Time-Domain Quasilinear Identification of Nonlinear Dynamic Systems

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A time-domain linear modal identification technique is applied to identify some highly nonlinear dynamic systems. The modal concept is used to identify such nonlinear systems with the understanding that the resulting modes are only a mathematical representation of the series solution of the nonlinear system under consideration. Naturally, these identified modal parameters are not unique for nonlinear systems, since they are functions of the systems' amplitudes and hence referred to as quasilinear. The approach presented in this paper can be useful in predicting signs of nonlinearities when linearity is assumed. It can also be used to analyze and understand the types of nonlinearities in nonlinear systems through successive identifications at different levels of response.

Nomenclature

f	= frequency, Hz
F_i	= force in restoring force element i
M	= mass
m	= number of degrees of freedom of the identification math model
n	= number of harmonics
$\{n(t)\}$	= measurement noise vector
N_i	= modal vector for noise representation
p	= number of degrees of freedom of system under identification
q	= number of degrees of freedom allowed for measurements noise
r_i	= ith characteristic root for noise representation
$\{x(t)\}$	= linear system response vector
$\{y(t)\}$	= nonlinear system response vector
$z(t)$	= displacement in restoring force element
α_i	= ith characteristic root for harmonics
Γ	= ith modal vector of harmonics
ζ_i	= ith damping factor, %
$\{\theta\}$	= angular displacement
$\{\psi_i\}$	= ith linear (or equivalent linear) modal vector
ITD	= Ibrahim time-domain modal identification technique
NMO	= number of modes allowed in the identification math model
SDOF	= single-degree-of-freedom system
TDOF	= two-degree-of-freedom system

Introduction

WITH the increasing complexity of modern aerospace and nonaerospace structures, accurate dynamic identification has become a necessity. Dynamic identification is usually carried out through identifying the structure's modal parameters. These modal parameters are required for modeling, responses and loads prediction, stability analysis, and control system design.

The demand for more sophisticated dynamic identification techniques, to match the stringent accuracy requirements for dynamic design and performance analysis, has resulted in numerous research efforts in this area during the last two decades.

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Presently, for dynamic identification a structural dynamicist has a choice between frequency-domain techniques[1,2] and time-domain techniques.[3-11] Although quite different and with merits that are still and will be debatable for some time, the two approaches have been used so far to deal only with linear systems. There have been a few efforts toward the dynamic identification of nonlinear dynamic systems.[12-19] Unfortunately, these efforts are limited to lumped parameter systems and are still academic and far from being applicable to real structures that possess some unknown forms of nonlinearity as well as an unknown number of degrees of freedom.

The assumption of linearity in dynamic identification has been found to be a reasonable one for many applications. This is true where amplitudes of vibration are small or when, in general, the levels of nonlinearities are small and can be ignored. On the other hand, some applications require serious consideration of their high-level nonlinearities where assuming linearity can be highly erroneous. An example of one such application is the case of the large-amplitude responses of panels subjected to acoustic and mechanical excitation.[20-26]

The purpose of this paper is first to study the applicability of a linear modal identification technique to nonlinear systems. The term "quasilinear" as used here is meant to perform the identification at one certain level of excitation or response. Although the modal approach may be nonexistent for a nonlinear system as a whole, the modal concept will be used here and it is understood that the identified modal parameters will be a function of the level of response of the system. The linear term "mode" is given to the harmonics as well as the fundamentals of the nonlinear system. Also, the linear term "model damping factor" is used to measure the systems harmonic coupling factors.

The second purpose of this paper is to identify the type of nonlinearities in the system. This can be attained by identifying the quasilinear modal parameters of the sytem at different levels of responses and studying the changes in these modal parameters.

The linear modal identification method selected here for the quasilinear identification of nonlinear systems is a time-domain one referred to as the Ibrahim time-domain (ITD) technique.[3-11] See the Appendix for an outline of the method.

It is to be noted here that, although the examples used in this paper are lumped parameter systems, the method is applicable as well to distributed parameter systems. The identification technique is not dependent on the number of degrees of freedom of the system under consideration. The identification model allows any larger number of degrees of freedom such that all modal information in the responses can be identified.

Theory: Linearized Identification Model

For linear systems, the ITD technique is based on the free-decay responses of a structure $\{x(t)\}$, which are linear combinations of the excited modes

$$\{x(t)\} = \sum_{i=1}^{2p} \{\psi_i\} e^{\lambda_i t} + \{n(t)\} \tag{1}$$

where $\{\psi_i\}$ is the ith modal vector, λ_i the ith characteristic root, p the number of modes excited in the responses ($2p$ complex conjugate modes), and $n(t)$ is the measurement noise.

The linear ITD technique also uses the concept of an oversized identification modal[11] to reduce the effects of measurement noise on the identified parameters. It was shown that allowing more degrees of freedom in the identification modal improves the accuracy of the identification, since the extra degrees of freedom act as outlets for the noise. Thus, Eq. (1) becomes

$$\{x(t)\} = \sum_{i=1}^{2p} \{\psi_i\} e^{\lambda_i t} + \sum_{k-2p+1}^{2m} \{N_k\} e^{r_k t} \tag{2}$$

where $m > p$.

This same concept can be used to develop a linearized identification model for nonlinear responses. The free-decay responses of a p-degree-of-freedom nonlinear system can be expressed as

$$\{y(t)\} = \sum_{i=1}^{2p} \{\psi_i\} e^{\lambda_i t} + \sum_{k=1}^{\infty} \{\Gamma_k\} e^{\alpha_k t} + \{n(t)\} \tag{3}$$

where in this case the first set of modes represents the fundamental solutions and the second set represents the harmonics.

If only a finite number of m degrees of freedom ($m > p$) are allowed in the identification model, then Eq. (3) becomes

$$\{y(t)\} = \sum_{i=1}^{2p} \{\psi_i\} e^{\lambda_i t} + \sum_{k=1}^{2n} \{\Gamma_k\} e^{\alpha_k t} + \sum_{\ell=1}^{2q} \{N_\ell\} e^{r_\ell t} \tag{4}$$

where

$$p + n + q = m$$

With the understanding that usually the amplitudes of higher harmonics get smaller for higher orders, the number of high harmonics to be identified will be dependent on the identification accuracy and levels of noises in the responses.

Applications

To test the validity and applicability of the preceding theory, the proposed approach is applied to three different nonlinear systems. These systems were selected to represent single-degree-of-freedom sytems with and without damping and a damped two-degree-of-freedom system. The nonlinear terms are restricted to the stiffness terms, while the damping terms were kept linear. For nonlinear stiffness, softening and hardening springs are represented. High nonlinearities were achieved by having the nonlinear term coefficient larger than that of the linear term and/or having large amplitudes of responses.

The simulated responses of these systems were obtained by numerically integrating the nonlinear differential equations with some specified initial displacements and zero initial velocities. A fourth-order Runge-Kutta with variable-step method was used for the numerical integration.

Simulated Systems

Three systems are simulated and identified using the preceding theory.

Simple Pendulum

The nonlinear differential equation of motion of the simple pendulum (Fig. 1) is

$$\ddot{\theta} + 4\pi^2 \sin\theta = 0 \tag{5a}$$

$$\ddot{\theta} + 0.04\pi\dot{\theta} + 4\pi^2 \sin\theta = 0 \tag{5b}$$

where Eq. (5a) represents the undamped case with a linear natural frequency of 1.0 Hz and Eq. (5b) has a 1.0% equivalent linear damping factor.

Responses were simulated for initial amplitudes θ_0 of $\pi/6$, $\pi/3$, $\pi/2$, and $2\pi/3$.

Mass-Spring System

A single-degree-of-freedom system (Fig. 2) was designed to have a hardening spring with an equivalent linear frequency of 1.0 Hz and equivalent linear damping of 1.0%. The governing equations of motion of such a system for undamped and damped cases are

$$\ddot{y} + 4\pi^2 (y + 2.0y^3) = 0 \tag{6a}$$

$$\ddot{y} + 0.04\pi\dot{y} + 4\pi^2 (y + 2.0y^3) = 0 \tag{6b}$$

Responses were simulated for initial amplitudes of 0.5, 1.0, 1.5, and 2.0 units.

Two-Degree-of-Freedom System

To simulate a multidegree-of-freedom nonlinear system with damping, a nonlinear system with two masses, three linear viscous dampers, and three nonlinear springs (Fig. 3) is analyzed. The two springs were selected with cubic nonlinearity representing hardening springs with the coefficients of the nonlinear term being 50 and 100% of that of

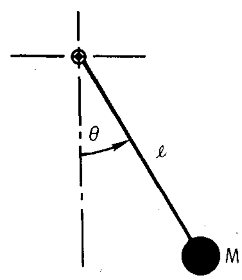

Fig. 1 Simple pendulum.

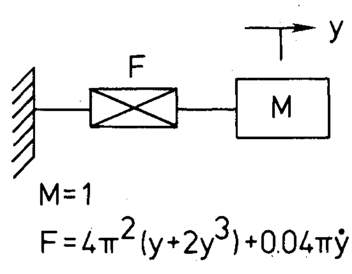

Fig. 2 Single-mass-spring system.

the linear term. The equation of motion of such a system are

$$\begin{Bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{Bmatrix} + \begin{bmatrix} 0.06\pi & -0.02\pi \\ -0.02\pi & 0.06\pi \end{bmatrix} \begin{Bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{Bmatrix} + \begin{bmatrix} 10\pi^2 & -6\pi^2 \\ -6\pi^2 & 10\pi^2 \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} + \begin{bmatrix} 7\pi^2 & -3\pi^2 \\ -3\pi^2 & 7\pi^2 \end{bmatrix} \begin{Bmatrix} y_1^3 \\ y_2^3 \end{Bmatrix} = \{0\} \quad (7)$$

Free responses due to initial displacements were obtained by numerically integrating Eq. (7). Two sets of initial conditions were used, the first to represent small amplitudes (0.3, 0.1) and the second to simulate larger amplitudes (3.0, 1.0).

Indentification

The numerically integrated responses were sampled at the rate of 50 Hz for the simple pendulum and the spring-mass system. Four seconds (200 samples) were used for the identification of both the SDOF ad TDOF systems. The number of modes allowed in the identification program was changed from 1 to 6 for the SDOF systems and from 1 to 12 for the TDOF system. Two samples were used to create the pseudomeasurements for the SDOF systems and six samples for the TDOF system. The parameter for delaying $[\hat{\phi}]$ from $[\phi]$ was taken as two samples for the simple pendulum and the TDOF systems and one sample for the spring-mass system. This means an aliasing frequency of 12.5 Hz for the first case and 25 Hz for the other. Not accounting for the errors arising from the numerical integration, cases with 0.0 and 1.0% noise/signal ratios were considered.

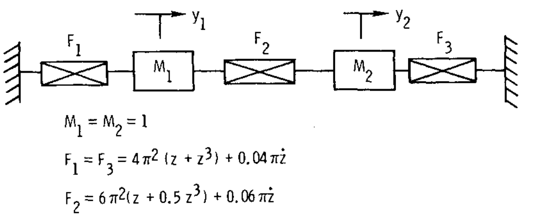

Fig. 3 Two-degree-of-freedom system.

Discussion

The Undamped Case (SDOF)

For the simple pendulum and single mass-spring system, the ITD was able to identify very accurately the fundamental frequency as well as the harmonics up to the ninth harmonic. Tables 1 and 2 list the identified harmonics for the two cases and Figs. 4 and 5 show these identified frequencies. Table 3 lists the identified damping factors for the undamped simple pendulum.

Tables 4 and 5 list the level of contribution of each harmonic showing the extremely small levels of contribution for higher harmonics.

Figures 6 and 7 show the identified fundamental frequencies as functions of the initial amplitude and their relation to the theoretical frequency of the nonlinear system. The same results are also shown in Tables 6 and 7.

As expected, more harmonics are identifiable for higher levels of nonlinearities. This also is evident from determinant plots shown in Figs. 8 and 9. For smaller initial amplitudes and smaller nonlinearities, the determinants decreased at a faster rate, indicating a lower level of harmonics.

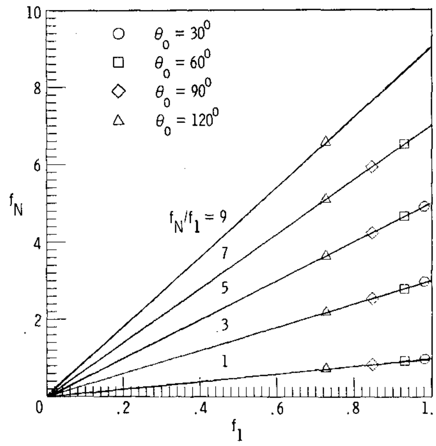

Fig. 4 Identified harmonics vs fundamental frequencies of simple pendulum.

Table 1 Ratios to fundamental of identified frequencies for undamped simple pendulum

Mode No., N	$\Phi_0=30$ deg f_N	$\Phi_0=30$ deg f_N/f_1	$\Phi_0=60$ deg f_N	$\Phi_0=60$ deg f_N/f_1	$\Phi_0=90$ deg f_N	$\Phi_0=90$ deg f_N/f_1	$\Phi_0=120$ deg f_N	$\Phi_0=120$ deg f_N/f_1
1	0.9829	1.000	0.9318	1.000	0.8472	1.000	0.7284	1.000
2	2.9846	3.937	2.7949	2.999	2.5417	3.000	2.1854	2.999
3	4.9162	5.002	4.6747	5.017	4.2360	5.000	3.6415	4.999
4	—	—	6.5269	7.005	5.9334	7.004	5.0987	7.000
5	—	—	—	—	—	—	6.5681	9.017
6	—	—	—	—	—	—	—	—

Table 2 Ratios to fundamental of identified frequencies for undamped spring-mass system

Mode No., N	$y_0=0.5$ f_N	$y_0=0.5$ f_N/f_1	$y_0=1.0$ f_N	$y_0=1.0$ f_N/f_1	$y_0=1.5$ f_N	$y_0=1.5$ f_N/f_1	$y_0=2.0$ f_N	$y_0=2.0$ f_N/f_1
1	1.1708	1.000	1.5691	1.000	2.0651	1.000	2.6032	1.000
2	3.5124	3.000	4.7073	3.000	6.1953	3.000	7.8116	3.001
3	5.8529	4.999	7.8456	5.000	10.3256	5.000	13.0204	5.002
4	—	—	10.9841	7.000	14.4605	7.002	18.2261	7.001
5	—	—	—	—	—	—	23.4437	9.006
6	—	—	—	—	—	—	—	—

Table 3 Identified damping factors[a] for undamped simple pendulum

	Damping factors, ζ_i				
θ_0, deg	1	2	3	4	5
30	0.00000	0.00065	0.00002	—	—
60	0.00001	0.00000	0.00050	−0.00096	—
90	0.00006	0.00001	0.00000	0.00122	—
120	0.00000	0.00002	0.00030	0.00866	0.00593

[a] All theoretical damping factors were equal to zero. The listed values were obtained from different identification runs for optimum damping identification.

Table 4 Amplitudes of identified harmonics for undamped simple pendulum

Initial amplitude, θ_0, rad		Identified $\theta_0 = \theta_1 + \theta_3 + \theta_5 + \theta_7 + \theta_9$				
Theory	Identified	θ_1	θ_3	θ_5	θ_7	θ_9
0.5236	0.5236	0.5244×10^0	-0.7474×10^{-3}	-0.1080×10^{-4}	—	—
1.0472	1.0472	0.1054×10^1	-0.6429×10^{-2}	0.6942×10^{-4}	-0.2919×10^{-6}	—
1.5708	1.5708	0.1594×10^1	-0.2396×10^{-1}	0.6300×10^{-3}	-0.1930×10^{-4}	—
2.0944	2.0944	0.2158×10^1	-0.6697×10^{-1}	0.3449×10^{-2}	-0.2058×10^{-3}	0.1156×10^{-4}

Table 5 Amplitudes of identified harmonics for undamped spring-mass system

Initial amplitude, y_0, rad		Identified $y_0 - y_1 + y_3 + y_5 + y_9$				
Theory	Identified	y_1	y_3	y_5	y_7	y_9
0.5000	0.5000	0.4942×10^0	0.5697×10^{-2}	0.6270×10^{-4}	—	—
1.0000	1.0000	0.9741×10^0	0.2529×10^{-1}	0.6399×10^{-3}	0.1577×10^{-4}	—
1.5000	1.5000	0.1449×10^1	0.4913×10^{-1}	0.1617×10^{-2}	0.6205×10^{-4}	—
2.0000	2.0005	0.1924×10^1	0.7323×10^{-1}	0.29871×10^{-2}	-0.3066×10^{-3}	0.7986×10^{-3}

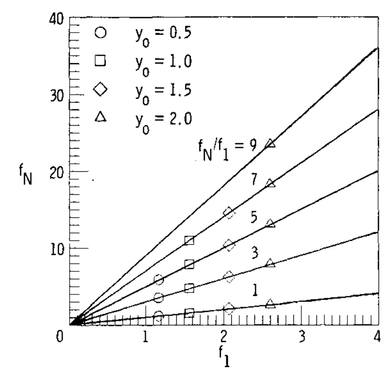

Fig. 5 Identified harmonics vs fundamental frequency of spring-mass system.

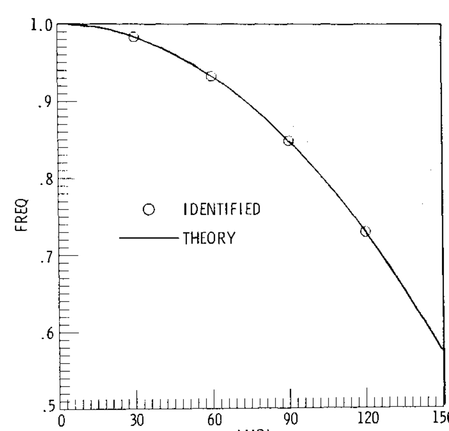

Fig. 6 Theoretical and identified fundamental frequency of simple pendulum.

Damped Case (SDOF)

The identification technique identified a strong fundamental frequency and very weak signs of a third harmonic. (See Tables 8 and 9.) Without noise added to the responses, early signs of singularities were noticed when the number of degrees of freedom was increased beyond two, an indication of extremely small nonidentifiable higher harmonics. Adding noise to the responses helps conditioning the response matrices and also masks the weakly excited harmonics.

For noise-free data, the larger number of degrees of freedom in the identification model revealed the changing frequency due to the decreasing amplitude of the response due to damping. The accuracy of the identification program detected the change in frequency between the measurement response and the pseudomeasurements. This phenomenon was not found when a small amount of noise (1.0%) was added to the response. This can also be avoided by using shorter time records for identification.

Two-Degree-of-Freedom System

Equations (7) were integrated with two sets of initial conditions, (0.3, 0.1) and (3.0, 1.0). These are only the initial conditions; responses in the second set had maximum displacements of 3.0 for both measurements.

Identification results showed nor more than two modes for the small-displacement case. For the larger-displacement case (higher nonlinearities), the two fundamental frequencies are higher than the linear case—a result that is expected from a system with hardening springs. Also, four other harmonics appeared in the identification output. Tables 10 and 11

Table 6 Theoretical and identified fundamental frequency for undamped simple pendulum

θ_0, deg	f_l, Hz Theory	Identified
30	0.9829	0.9829
60	0.9318	0.9318
90	0.8472	0.8472
120	0.7284	0.7284

Table 7 Theoretical and identified fundamental frequency for undamped spring-mass system

y_0, rad	f_l, Hz Theory	Identified
0.5	1.1708	1.1711
1.0	1.5691	1.5693
1.5	2.0651	2.0650
2.0	2.6032	2.6042

Table 8 Identified frequencies and damping factors for simple pendulum

θ_0, deg	f, Hz	ζ, %
30	0.9906	0.93
	2.9186	3.50
60	0.9398	1.15
	2.7712	2.89
90	0.8678	1.30
	2.5698	2.60
120	0.7714	1.89
	2.2955	4.67

Table 9 Identified frequencies and damping factors for spring-mass system

y_0, rad	f, Hz	ζ, %
0.5	1.1542	0.76
	3.4240	2.11
1.0	1.5242	0.49
	4.5873	1.98
1.5	1.9935	0.36
	6.0214	1.89
2.0	2.5064	0.33
	7.5700	1.58

Table 10 Identified frequencies and damping factors for the two-degree-of-freedom system[a]

y_{01},	y_{02}	Mode	f, Hz	ζ, %
0.3,	0.1	1	1.0185	0.95
		2	2.0359	0.99
3.0,	1.0	1	2.2783	1.40
		2	4.0214	1.34
		3	5.7955	1.07
		4	8.5792	0.65
		5	10.4157	1.47
		6	12.1253	1.60

[a]Theoretical linear frequencies are 1.0 and 2.0 Hz and damping factors are 1.0% for the two modes.

summarize the identified quasilinear modal parameters for the system. Table 11 shows that, for the large-amplitude case, the first two modes have the largest contributions to the responses, also indicating that these two modes are the fundamental ones.

For a better understanding of the results in Table 11, the linear modes of the system would have resulted in modal contribution vectors of {0.2 0.2} and {0.1 −0.1} for the small-amplitude case and {2.0 2.0} and {1.0 −1.0} for the larger-amplitude case. For the nonlinear identified modal vectors, the amplitudes of the fundamental modes did not

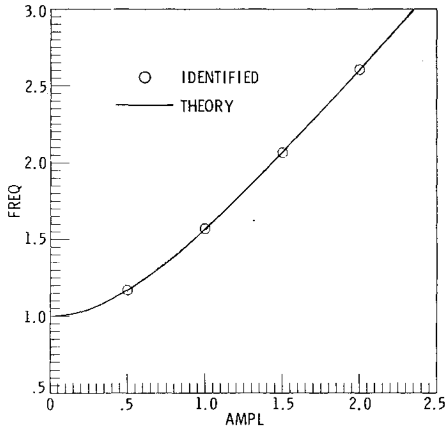

Fig. 7 Theoretical and identified fundamental frequency of spring-mass system.

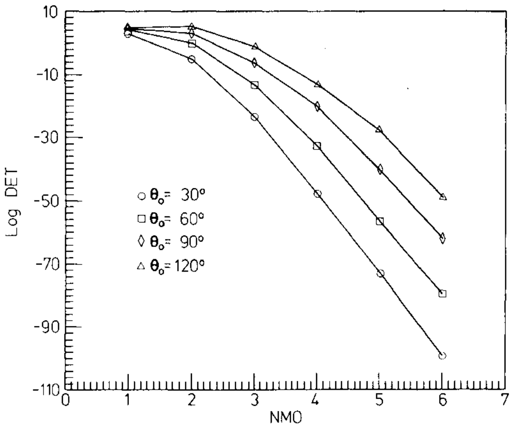

Fig. 8 Determinant vs DOF of identification model for simple pendulum.

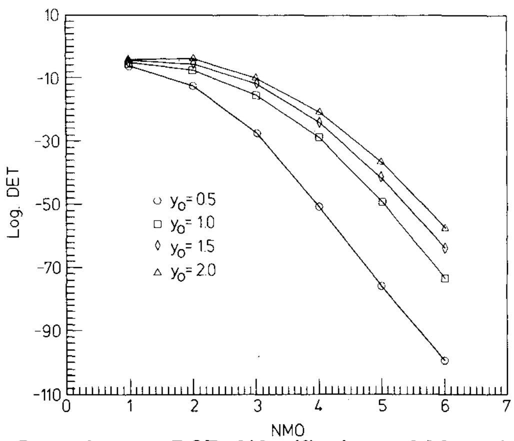

Fig. 9 Determinants vs DOF of identification model for spring-mass system.

Table 11 Identified quasilinear mode shapes for the two-degree-of-freedom system

Case (y_{01}, y_{02})	Station	Mode No. 1	2	3	4	5	6
0.3, 0.1	1—Amplitude	0.1992	0.10156				
	phase, deg	0.21	0.23				
	2—Amplitude	0.1995	0.1014				
	phase, deg	0.25	178.84				
3.0, 1.0	1—Amplitude	2.0408	1.7412	0.0643	0.1191	0.0455	0.0578
	phase, deg	−8.10	−29.30	−106.00	−71.94	−94.91	−126.47
	2—Amplitude	1.9648	1.8263	0.0828	0.1121	0.0333	0.0547
	phase, deg	−11.45	99.10	−154.79	−96.12	−35.31	78.94

change much from the linear amplitudes, but large changes in the phase angles occurred.

Conclusion

The time-domain, linear modal identification technique is found to be useful for the quasilinear modal identification of nonlinear dynamic systems. Such approach can be used to detect nonlinearities and their types in structures by performing the identification at different levels of response and to study the changes in the identified modal parameters.

Appendix: Background of ITD Modal Identification Technique

The ITD modal identification technique is based on the fact that a vector of free-decay responses of $2m$ measurements and/or pseudomeasurements, containing contributions from p modes, can be expressed as

$$\{\phi(t)\} = \sum_{i=1}^{2p} \{\psi_i\} e^{\lambda_i t} \tag{A1}$$

To allow using an identification mathematical model with an m degrees of freedom, where $m \gg p$, the vector $\{\phi(t)\}$ is arbitrarily selected as

$$\left\{\begin{array}{l} \{x(t)\} \\ \{x(t+\Delta t_2)\} \\ \{x(t+2\Delta t_2)\} \\ \vdots \\ \{x(t+\Delta t_3)\} \\ \{x(t+\Delta t_2+\Delta t_3)\} \\ \{x(t+2\Delta t_2+\Delta t_3)\} \\ \vdots \end{array}\right\} \tag{A2}$$

where $\{x(t)\}$ is the measured response vector. Equation (A1) can be written for the same responses delayed Δt_l in time as

$$\{\hat{\phi}(t)\} = \{\phi(t+\Delta t_l)\} = \sum_{i=1}^{2p} \{\psi_i\} e^{\lambda_i \Delta t_l} \cdot e^{\lambda_i t} \tag{A3}$$

By repeating these measurement vectors $\{\phi\}$ and $\{\hat{\phi}\}$ for $2r$ times (where $r > m$) to form the two $2m \times 2r$ response matrices $[\phi]$ and $[\hat{\phi}]$, and computing a matrix $[A]$ for which one possible solution is satisfied by

$$[A][\phi][\phi]^T = [\hat{\phi}][\phi]^T \tag{A4}$$

the modal parameters of the system under consideration can be determined from the eigenvalue problem,

$$[A]\{\psi_i\} = e^{\lambda_i \Delta t_l}\{\psi_i\} \tag{A5}$$

For complete details on the ITD technique, see Refs. 3-11.

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References

[1] Klosterman, A. and Zimmerman, R., "Modal Survey Activity Via Frequency Response Functions," SAE Paper 751068, Society of Automotive Engineers, 1975.

[2] Brown, D. L., Allemang, R. J., Zimmerman, R., and Mergeay, M. "Parameter Estimation Techniques for Modal Analysis," SAE Paper 790221, Society of Automotive Engineers, 1979.

[3] Ibrahim, S. R. and Mikulcik, E. C., "The Experimental Determination of Vibration Parameters from Time Responses," *Shock and Vibration Bulletin*, No. 46, Pt. 5, Aug. 1976, pp. 187-196.

[4] Ibrahim, S. R. and Mikulcik, E. C., "A Method for the Direct Identification of Vibration Parameters from the Free Response," *Shock and Vibration Bulletin*, No. 47, Pt. 4, Sept. 1977, pp. 183-198.

[5] Ibrahim, S. R., "Random Decrement Technique for Modal Identification of Structures," *Journal of Spacecraft and Rockets*, Vol. 14, Nov. 1977, pp. 696-700.

[6] Ibrahim, S. R., "Modal Confidence Factor in Vibration Testing," *Journal of Spacecraft and Rockets*, Vol. 15, Sept.-Oct. 1978, pp. 313-316.

[7] Ibrahim, S. R., "Application of Random Time Domain Analysis to Dynamic Flight Measurements," *Shock and Vibration Bulletin*, No. 49, Pt. 2, Sept. 1979, pp. 165-170.

[8] Hanks, B. R., Miserentino, R., Ibrahim, S. R., Lee, S. H., and Wada, B. K., "Comparison of Modal Test Methods on the Voyager Payload," SAE Paper 781044, Society of Automotive Engineers, Nov. 1978.

[9] Ibrahim, S. R. and Mikulcik, E. C., "Time Domain Identification of Standing Wave Parameters in Gas Piping Systems," *Journal of Sound and Vibration*, Vol. 60, Pt. 1, pp. 21-31.

[10] Pappa, R. S. and Ibrahim, S. R., "A Parametric Study of the Ibrahim Time Domain Modal Identification Algorithm," *Shock and Vibration Bulletin*, No. 51, June 1981.

[11] Ibrahim, S. R. and Pappa, R. S., "Large Modal Survey Testing Using the Ibrahim Time Domain (ITD) Identification Technique," *Journal of Spacecraft and Rockets*, Vol. 19, Sept.-Oct. 1982, pp. 459-465.

[12] Beliveau, J. G., "Identification of Viscous Damping in Structures from Model Information," *Journal of Applied Mechanics*, Vol. 43, 1976, pp. 335-339.

[13] Berman, A., "System Identification of a Complex Structure," AIAA Paper, May 1975.

[14] Collins, J. D., et al., "Statistical Identification of Structures," *AIAA Journal*, Vol. 12, 1970, pp. 185-190.

[15] Distefano, N. and Todeschini, R., "Modeling, Identification and Prediction of a Class of Nonlinear Viscoelastic Materials," *International Journal of Solids and Structures*, Vol. 9, Pt. 1, 1973, pp. 805-818.

[16]Udwadia, F. E. and Shah, P. C., "Identification of Structures Through Records Obtained During Strong Earthquake Ground Motion," *Journal of Engineering for Industry*, Vol. 98, No. 4, 1976, pp. 1347-1362.

[17]Masri, S. F. and Caughey, T. K., "A Non-Parametric Identification Technique for Non-Linear Dynamic Problems," *Journal of Applied Mechanics*, Vol. 46, 1979, pp. 433-447.

[18]Udwadia, F. E. and Kuo, C. P., "Non-Parametric Identification of a Class of Non-Linear Cross-Coupled Dynamic Systems," *Earthquake Engineering and Structural Dynamics*, Vol. 9, 1981, pp. 385-409.

[19]Yang, Y. and Ibrahim, S. R., "A Nonparametric Identification Technique for a Variety of Discrete Nonlinear Systems," *ASME Journal of Vibration, Acoustics, Stress & Reliability in Design*, to appear.

[20]Holehouse, I., "Sonic Fatigue Design Techniques for Advanced Composite Aircraft Structures," AFWAL-TR-80-3019, April 1980.

[21]Mei, C., "Large Amplitude Response of Complex Structures Due to High Intensity Noise," AFFDL-TR-79-3028, April 1979.

[22]Mei, C., "Response of Nonlinear Structural Panels Subjected to High Intensity Noise," AFWAL-TR-80-3018, March 1980.

[23]Mei, C. and Wentz, K. R., "Large Amplitude Random Response Angle-Ply Laminated Composite Plates," *Proceedings AIAA Dynamics Specialists Conference*, Atlanta, Ga., April 1981, pp. 559-573.

[24]Chu, H. N. and Herrmann, G., "Influence of Large Amplitudes on Free Flexural Vibrations of Rectangular Elastic Plates," *Journal of Applied Mechanics*, Vol. 23, Dec. 1956, pp. 532-540.

[25]Mei, C. and Wentz, K. R., "Analytical and Experimental Nonlinear Response of Rectangular Panels to Acoustic Excitation," *Proceedings of AIAA Structures, Structural Dynamics and Materials Conference*, New Orleans, La., May 1982, pp. 514-520, AIAA Paper 82-0733.

[26]White, R. G., "Comparison of the Statistical Properties of the Responses of Aluminum and CFRP Plates to Acoustic Excitation," *Journal of Composites*, Oct. 1978, pp. 251-258.